SYNOPSIS

'Characterization of Materials in some Structural Components'

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Session: 2018-19

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1. INTRODUCTION:

Mechanics of Solids deals with the mechanical behavior of the deformable bodies (bars, plates, cylinder, disc, shell etc.) subjected to various types of external forces and goes by a variety of names, such as strength of materials, mechanics of deformable solids etc. Research in solid mechanics is not only for the basic understanding of mechanical phenomenon but also for the advanced engineering techniques in most of the areas throughout mechanical and structural technology. Advances in the subject are central to assure safety, reliability and economy in the design of device, structure and complete systems, and hence to the continued development of power generation technology such as nuclear fusion and gas turbines, aerospace and surface transportation vehicles, earthquake resistant design, offshore structures, orthopedic devices, material processing and manufacturing technologies.

Recent developments in the field of solid mechanics, which has space and nuclear engineering as its ruling parts has required not only the exploitation of superior high temperature alloys needed for many modern construction such as the hypersonic planes, jet engines, missiles, gas turbines, nuclear reactors etc. which endure to severe conditions of high temperature and high stress, but has also required the research of analytical methods of stress and strain for various environments becomes an important engineering. Considerations of plastic state of matter are today of interest to many branches of sciences and engineering. The material engineers are interested in plastic flow as it relates forming processes such as rolling, forging, bending, stretching and deep drawing. The design engineer is interested in deformation behaviors in connection with his responsibility for avoiding excessive deflection or distortion in machine parts and engineering structures. The steel and metal worker desires to control more accurately the mechanical process of forming metals at forging temperature. Because of large quantity of energy, at present time, consumed in steel mills during the process of rolling, a more economical use of energy is needed. On the other hand, in order to choose the right material for various parts of his machine, the machine designer must carefully consider the mechanical properties of these materials. He is not only interested in a more exact knowledge of the limiting conditions of stress at which, in his machine parts, permanent set begins to develop and danger of yielding of fracture is to be expected of fatigue cracks begin to form but also in several cases he will have to consider the possible deformation in

machine parts exposed to long duration of stressing. He may base his calculation as far as they refer to purely elastic deformation upon the theory of elasticity, but he lacks in the theory of strength of materials and similarly certain basis when considering the transition from elastic to plastic deformation or the condition of rupture. Therefore, the theory of plasticity is made necessary by demand for better design and more economic use of materials and for better control of metal working operations such as wire-drawing, cold rolling and extrusion.

Elasticity: Elasticity deals with the systematic study of stress, strain and displacement in an elastic body under the influence of external forces. All structural materials posses to a certain extent the property of elasticity, that is, if external forces producing deformation of a structure do not exceed to a certain limit, the deformation disappears with the removal of the forces. For various engineering disciplines the purpose of studying elasticity is to analyze the stresses and displacements of structural and machine elements in the range and thereby to check the sufficiency of their strength, stiffness and stability.

Plasticity: Plasticity is the name given to the mathematical study of stress and strain in plastically deformed solids. This follows the well-established precedent set up by the 'theory of elasticity'. The relation of plastic and elastic properties of metals to crystal structures and cohesive forces belongs to the subject now known as 'metal physics'. The theory of plasticity takes, as its starting point, certain experimental observations of the macroscopic behaviors of a plastic solid in uniform states of combined stress. The task of the theory is two fold: First, to construct explicit relations between stress and strain agreeing with the observations as closely and as universally as needed, and second, to develop mathematical techniques for calculating non-uniform distributions of stress and strain in bodies permanently distorted in any way. Unlike elastic solids in which the state of strain depends only on the final state of stress, the deformation that occurs in plastic solid is determined by the complete history of loading. The plasticity problem is, therefore, essentially incremental in nature, the final distortion of the solid being obtained as the sum total of the incremental distortions following the strain path. The situation in which material deforms continuously to load for a prolonged period of time, usually at elevated temperature, is called 'creep' and so a constant load test is called a 'creep test'. The conventional stress (load divided by initial cross-section) is called the 'creep-stress'. The gradual strain is called 'creep-strain', the strain-time curve one obtains is called

the ‘creep-curve’ and the slope of this curve is called the ‘creep-rate’. Creep strains may be elastic, plastic or a combination of these. Creep strain might occur too slowly to detect or too rapidly to follow. Even in such cases, however, there is evidence that creep strain is always time-dependent. For many material including most metals and alloys, elevated temperatures may be applied in order for creep to be produced. Changes in temperature induce thermal expansion and have ill-defined effect upon the material behavior. However, in various non-metallic materials such as plastics, wood and concrete, creep occurs at normal temperatures.

Transition: Transition is a natural phenomenon and there is hardly any branch of science or technology in which we do not come across transition from one state in to another. Elasticity-plasticity, visco-elastic, deformations, creep, fatigue relaxation and adiabatic theory in quantum mechanics is well known examples. At present they are generally treated as discontinuous. For elastic-plastic deformations, the ideal model has to be assumed for some type of yield condition. For creep, not only the yield condition but also a number of creep strain laws have to be assumed. All these assumptions leave the mind a little dissatisfied. More scientific advancements should attempt to reduce these assumptions. This has motivated us to pay some attention to the treatment of transition problems in the domain of solid mechanics.

The deformation theory of plasticity does not satisfy the condition of continuity of the relationship between stress and deformation on transition from loading to unloading. In analyzing this and other theories of plasticity we may encounter similar circumstances in which certain consequences of the theory appear to be contradictory or physically unacceptable. In the classical theory of plasticity, the material region is assumed to be divided into elastic and plastic regions, which are separated by an yield surface depending on the symmetry and other physical considerations. In the other words perfect elasticity and ideal plasticity are two extreme properties of the material and the use of ad-hoc rule like yield condition amounts to be divide the two extreme properties by a sharp line, which is not physically possible. In the elastic region, classical theory of linear elasticity holds. In the plastic, region Prandtl-Reuss or Von-Mises equations are used with a yield condition and the boundary condition that the normal stress components should be continuous on the yield surface. This linear theory has given rise to a vast amount of important results, which have been corroborated by experiments more exactly, than one may expect. However, its main drawbacks are the following assumptions:

1. That even through the material at a point has yielded; the material at a neighbouring point still remains elastic.
2. That an yield surface of the assumed type separating the elastic and plastic regions exists.
3. That for any given material there is a function of the three principal stresses which always has a value when yielding begins regardless of the stress state; and
4. That the same functional relationship applies to all materials although the numerical value of the function is different for different materials.

Theory of elasticity and plasticity describe the mechanics of deformation of most engineering solids. Both the theories as applied to metals and alloys are based on experimental studies of the relation between stress and strain in a polycrystalline aggregate under simple loading conditions. Thus they are of phenomenological nature on the macroscopic scale and, as yet, owe little to knowledge of structure of metals. However, in order to understand the limitations so imposed on the theories, the engineer, with his main interest in design and manufacture must have some knowledge of the structure of metals. The validity of the predictions of any mathematical model of deformation and stress-distribution no matter how carefully constructed depends ultimately on one's ability to assess the mechanical properties and characteristic of the material to be used and to apply these data in the actual calculations. For this reason, testing material properties is a matter of utmost importance for an engineer. No design can possibly be regarded as successful unless it is firmly based on good understanding of the potential of the material and its consequent ability to respond satisfactorily to the proposed system of loading. It is important to realize that any mechanical testing technique no matter how appropriate can provide information only about the average material properties to be generally expected but can not provide an explanation of why a material behaves in a certain manner. It does not even necessarily follow that data obtained in a test on a specific material will be totally reliable when the material ostensibly the same, but from a different batch, is used in the manufacturing process. We can only expect that in the absence of any unknown internal defects and metallic and non-metallic inclusions. Our alloy will behave in the way suggested by the test results. The basic method for bringing out the behaviour of structural materials is the tensile test.

Elastic-Plastic transition is obtained in current literature with the help of semi-empirical yield condition like that of Tresca or von-Mises. The stresses are obtained from the elastic solution and then substituted in the yield condition to get the transition surface. The possibility of treating it as a transition or turning point phenomenon in finite deformation has not been explored. When plastic state tends to set in, the stress-strain relation undergoes a change. This must be reflected in our equations. The linear classical theory cannot do it. Tresca (1868) pointed out that a transition state, which he called mid-zone exists when a material passes from elastic - zone to plastic - zone and this is, later on supported by Todhunter and Pearson (1893). In the transition state whole of the material participates, and not simply a selected region or a line as assumed by classical theories. This is expressed by Sir Lawrence and supported by Bragg (1948). When he points out that an element of a crystal embedded in the matrix shall be pictured as being strained together with the whole mass of the metal and hence slip must take place by a whole intra-atomic distance.

The demand of high-speed technology in transportation, communication and energy conversion have forced us to take serious notice of non-linearity. But some of us still find it difficult to get rid of the habit of a century old habit of analytical and experimental research in continuum mechanics, which leisurely leaned very heavily on linearization. If a medium ‘A’ changes into ‘B’ through a transition state T, ‘A’ and ‘B’ may be almost linear but T is non - linear. Since this non - linearity is difficult to investigate, workers have taken to the artifice of replacing it by singular, non-differentiable or discontinuous surfaces. This piece-wise treatment necessitates the use of ad-hoc and semi-empirical laws, which may or may not exist. Linearizing non-linear problems by perturbation, boundary layer and other techniques do not provide satisfactory explanation for some important characteristics of non-linearity. As a result a number of important physical effects do not get adequate scientific expression. There is hardly any information, which we do not know about any linear fields. Their existence, uniqueness and stability are well established. For elastic-plastic deformation, the ideal model has to be assumed for some type of yield condition. For creep, not only the yield condition but also a number of creep strain law has to assumed. All these assumptions leave the mind a little dissatisfied. Any scientific advance should attempt to reduce these assumptions. This has motivated us to pay some attention to the treatment of transition problems in the domain of solid

mechanics. Science of elasticity is based on the fundamental notation of stress and strain and subjected to a restriction on their magnitude on the proportionality between them know as Hooke's law. The precise definition of stresses and strain is thus a matter of fundamental importance. Such definitions have necessarily to be comprehensive. They have to cover cases where their magnitudes vary from point to point within the crystal and also cases in which the stresses and strain vary with times as in the theory of wave propagation.

2. LITERATURE REVIEW:

Disc: Rotating disc play an important role in machine design. The design of elastic-plastic rotating discs has been a subject of longstanding interest with useful and practical applications in industry and engineering such as turbine rotors, compressors, flywheels, high speed gear engines and computer devices etc. The problem of rotating discs was first treated in the early nineteenth century. Solution of the isotropic discs including variable thickness, variable density and other case can be found in most of the standard elasticity textbooks (Arrowsmith, 1923; Heyman, 1958). Rotating discs were considered to be one of the exhausted subjects in the field of solid mechanics. Recently new interest has been generated to reinvestigate this centuries old problem. The new interest is the result of the development of composite materials and their applications. Composite materials are characterized by high strength to weight ratios, heterogeneity and anisotropy characteristics. Thus rotating disc technique has been proved to be a simple and reliable means of generating a biaxial state of stress when the loads cannot be directly applied to the material under investigation. In this case, an analytical solution of this problem is needed to interpret the experimental results generated in the laboratory. Because of the steady increase in the use of energy and the impact of that use on the environment, a flywheel made of composite materials has been proved to be an efficient mean of storing energy. Likewise, analytical solution of rotating discs made of composite materials is essential in design and analysis. The calculation of turbine disc involves the solution of second order differential equation having the elastic stress function. This equation is integrable analytically for discs of constant thickness, and for specific examples of discs of non-constant thickness. For example, Arrowsmith (1923) has given solutions for discs whose thickness is inversely proportional to radius to the power thickness

parameter, while Malkin (1934) discusses exponential disc. For a disc of non-uniform profile, however, the fact that the hoop stress at the hub does not exceed the permissible value tells the designer little more than that the disc is safe at the design speed. One cannot, from this knowledge, alone calculate the percentage over-speed at which the disc will burst or more realistically, at which the diameter of the disc will increase sufficiently to strip the blades. The safety factor on stress is thus replaced by a safety factor on the speed. Attention has been paid recently to the calculation of bursting speed, usually of flat discs made from strain-hardening material. This is of great interest from the stand point of plastic analysis, but the designer is usually more concerned with the speed at which large deformations brings, that is, in flow speed. Unrestricted plastic flow is an unstable plastic flow for a disc unless the material strain-hardens as soon as the whole disc becomes plastic, the radial displacement increases rapidly and the disc bursts. The calculation of the condition at which large displacements begin is likely to be of most interest to the designer.

Cylinder: Circular cylinders subjected to high internal pressure are widely used in various industries. In general, vessels under high pressure require a strict analysis for an optimum design for reliable and secure operational performance. Efforts were continually made to increase reliability. Solutions have been obtained either in analytical form or with numerical implementations. The creep analysis of thick-walled cylinder under internal pressure has been investigated by Weir (1957)and King *et.al.*(1967) for cylinders of isotropic materials. Rimrott (1959) has solved the problem of thick-walled isotropic cylinders subject to internal pressure using finite strain theory and has shown that by considering finite strains the creep rate of a thick walled cylinder subject to internal pressure increases even though the creep rate of the same material when subjected to constant true stress in simple tension is constant.

Shell: Rotating shell structures have many engineering applications like aviation, rocketry, missiles, electric motors and locomotive engines. Engineers have found its increasing application in aerospace, chemical, civil and mechanical industries such as in high-speed centrifugal separators, gas turbines for high-power aircraft engines, spinning satellite structures, certain rotor systems and rotating magnetic shields. To increase the strength of shells or shafts, it is, therefore, very important for engineers to study the behavior of transition of rotating shells. Solutions for thin spherical shells can be found in most of the standard elasticity and plasticity textbooks Timoshenko

and Goodier(1970), Chakrabarty (1987). Elastic behavior of shells has been very closely investigated, mostly by means of the finite element method. Many authors like Eberlein (1999), Civalek et al. (2009) have done elastic-plastic calculations in shells by using the various theoretical and numerical approaches based on finite element method, shear deformation theory, discrete convolution technique Schmidt and Weichert (1989), Simo et al. (1990), Eberlein and Wriggers (1999), Civalek and Gürses (2009). A shell is a curved surface, in which the thickness is much smaller than the remaining dimensions. The geometrical properties of shells, i.e. single or double curvature give rise to a tremendous advantage of these light-weight structures Woelke (2005). Many author worked on the problems of disc, cylinder and shell as below:

Hulsurkar Suresh (1979) have analyzed thermo elastic plastic transition in rotating disks with steady state temperature. It has been observed that if the additional condition of incompressibility is imposed, then the expression for stresses correspond to those arising from Tresca yield criterion.

Gupta S. K., Shukla R. K. (1994) have analyzed the effect of non-homogeneity on elastic-plastic transition in a thin rotating disc. It has been observed that in the presence of non-homogeneity ($k > 0$) in thin rotating discs required higher angular velocity for initial yielding as compare to homogeneous disc but less percent increase in angular velocity to become fully plastic against initial yielding and this percentage goes on decreasing with the increase in non-homogeneity. Reverse is the case for $k < 0$.

Shukla R. K (1996): have determined creep stresses in a thin rotating annular disc made of non-homogeneous material by using transition theory. It has been observed that in the presence of non-homogeneity having lesser value at the bore, reduces the stresses and the angular velocity required for steady state of creep as compared to the homogeneous disc, however, that having higher value at the bore than at the rim, increasing the magnitude of angular velocity and creep stress significantly, and hence increases the possibility of a fracture in the vicinity of the bore.

Güven U. (1997) have analyzed fully Plastic rotating disk with rigid inclusion. The plane state of stress in a fully plastic rotating annular disk mounted on a rigid circular shaft is studied. The analysis is based on Tresca's yield condition, its associated flow rule, and linear strain hardening material behaviour.

Güven U. (1998) has obtained elastic-plastic stress distribution of a rotating hyperbolic annular disk mounted on a circular rigid shaft. The exact solution presented is based on the usual assumptions of plane stress, Tresca's yield condition, its associated flow rule and linear strain hardening material behaviour. According to the present analysis the plastic core consists of three parts with different forms of the yield condition. The theory presented is illustrated by numerical results. The results show that the plastic flow and maximum stresses are influenced by the thickness parameter.

Güven U. (1999) has analyzed plane state of stress in an elastic-plastic rotating annular disk mounted on a rigid circular shaft. The analysis is based on Tresca's yield condition, specifically its associated flow rule and linear strain hardening material behavior. In the present analysis, it is shown that the plastic core adopts three parts, within different forms of the yield condition.

Gupta S. K. et al. (2000) have analyzed creep stresses and strain rates in a thin rotating disc having variable thickness and variable density by using Seth's transition theory. It has been concluded that a rotating disc whose density and thickness ratio decrease radially is on the safer side of the design in comparison to a flat disc having variable density.

Shukla R. K. (2000) has obtained elastics-plastic transitional stresses in a non-homogeneous disc with variable thickness subjected to internal pressure by using Seth's transition theory. Non-homogeneity in the disc is due to variation of Poisson's ratio of the material. It is found that the presence of non-homogeneity and thickness variation influence significantly the stresses and pressure required for initial yielding. The thickness variation reduces the magnitude of the stresses and pressure needed for fully-plastic state.

Ahmet et al. (2004) have analyzed parametric analysis of rotating variable thickness elastoplastic annular disks with inner boundaries subject to pressure or mounted on rigid inclusion. A computer model based on von Mises yield criterion, J2-deformation theory and nonlinear isotropic hardening is developed. Elastic limit angular velocities, partially plastic deformations and plastic limit angular velocities are investigated with emphasis on the determination of the effects of all the geometric, material and hardening parameters involved.

Gupta S.K., Thakur P.(2007)have analyzed rotating disc with inclusion and made of compressible material requires lesser angular speed to yield at the internal surface and

higher percentage increase in angular speed to become fully plastic as compare to disc made of in compressible material. With effect of thermal rotating disc required lesser angular speed to yield at the internal surface. Rotating disc made of compressible material with inclusion requires higher percentage increase in angular speed to become fully-plastic as compare to disc made of in compressible material. Thermal effect also increases the values of radial and circumferential stresses at the internal surface for fully-plastic state.

Gupta S. K., Thakur P. (2007) have obtained creep stresses and strain rates for a thin rotating disc with inclusion by using Seth's transition theory. It has been observed that radial stress has maximum value at the internal surface of the rotating disc made of incompressible material as compared to circumferential stress and this value of radial stress further increases with the increase in angular speed. Strain rates have maximum values at the internal surface for compressible material. Rotating disc is likely to fracture by cleavage close to the inclusion at the bore

Thakur P. (2009) have obtained elastic - Plastic Transition Stresses in a transversely isotropic thick -walled cylinder subjected to internal pressure and steady - state temperature by using transition theory. It has been seen that with the increase of temperature, the cylinder having smaller radii ratios requires lesser angular velocity to become fully plastic as compared to cylinder having higher radii ratios

Thakur P. (2009) has analyzed a disc of variable thickness made of compressible material yields at some radius R_1 at a higher pressure as compared to disc made of incompressible material that yields at the outer surface. Flat disc made of incompressible material yields on internal surface at higher pressure as compared to disc made of compressible material. Circumferential stress is maximum at the outer surface of the disc having variable thickness.

Thakur P. (2009) has performed elastic-plastic transition in a thin rotating disc having variable density with inclusion using Seth's transition theory. It has been observed that the rotating disc with inclusion and made of compressible material yields at the internal surface at a lesser angular speed as compared to a disc made of incompressible material whereas it requires a highly percentage increase in angular speed to become fully plastic. Radial stresses are higher at the bore for rotating disc made of incompressible material. It means that rotating disc having variable density and made of compressible material have a tendency to fracture at the bore. Effect of

variable density also increases the values of radial and circumferential stresses at the internal surface for fully-plastic state.

Thakur P. (2010) has obtained stresses for the elastic-plastic transition and fully plastic state for a thin rotating disc with rigid shaft at different temperatures. It has been observed that at room temperature rotating disc made of compressible material and of smaller radii ratio yields at the internal surface at a higher angular speed as compared to rotating disc made of incompressible material. With the introduction of thermal effect rotating disc yields at the outer surface at a lesser angular speed as compared to rotating disc at room temperature. The circumferential stress is maximum at the outer surface of the rotating disc with further increases with the increase in temperature. It means that angular speed of the rotating disc is less than that of the temperature-loaded disc in the fully plastic case.

Thakur P. (2010) has calculated creep stresses and strain rates for a thin rotating disc with shaft at different temperature. It has been observed that radial stress has maximum value at the internal surface of the rotating disc made of incompressible material as compared to circumferential stress and this value of radial stress further increase with the increase of angular speed. With the introduction of thermal effect, it has been observed that radial stress has higher maximum value at the internal surface of the rotating disc made of incompressible material as compared to circumferential stress, and this value of radial stress further increases with the increase of angular speed as compared to the case without thermal effect. Strain rates have maximum values at the internal surface for compressible material. Rotating disc is likely to fracture by cleavage close to the inclusion at the bore.

Thakur P. (2011) analyzed the problem of creep Transition stresses of a Thick isotropic spherical shell by finitesimal deformation under steady state of temperature and internal pressure. It is seen that shell made of incompressible material require higher pressure to yield as compared to shell made of compressible material.

Thakur P. (2011) Elastic-plastic Transition stresses in rotating cylinder by finite deformation under steady-state temperature. It has been seen that with the increase of temperature, the cylinder having smaller radii ratios requires lesser angular velocity to become fully plastic as compared to cylinder having higher radii ratios.

Thakur P. (2011) have analyzed elastic - plastic transitional stresses in an annular disc having variable thickness and variable Poisson's ratio subjected to internal pressure using Seth's transition theory. It is seen that thickness and passion's

ratio variation influence significantly the stresses and pressure required for initial yielding. The thickness variation reduces the magnitude of the stresses and pressure needed for fully plastic state. It is seemed for fully plastic state that circumferential stresses is maximum at the outer surface.

Thakur P. (2012) have obtained stresses for the elastic-plastic transition and fully plastic state for a thin rotating disc with rigid shaft having variable thickness by using Seth's Transition theory . It has been observed that in the absence of thickness, rotating disc with inclusion and made of compressible material e.g. Copper, Brass and Steel, yields at the internal surface at a lesser angular speed as compared to a rotating disc made of incompressible material e.g. rubber whereas it requires a higher percentage increase in angular speed to become fully plastic. With the effect of variation thickness, higher angular speed is required to yield at the internal surface. It has been observed that the radial stress has maximum at the internal surface. With the effect of variable thickness it increases the value of radial and circumferential stress at the internal surface for transitional state, whereas it can be seen that rotating disc having variable thickness increases the values of radial and circumferential stress at the internal surface for fully-plastic state.

Thakur P. (2012) has derived the elastic-plastic and transitional stresses using transition theory. It is observed that the rotating disc made of incompressible material with inclusion requires higher angular speed to yield at the internal surface as compared to the disc made of compressible material. It is seen that the radial and circumferential stresses are maximum at the internal surface with and without edge load (for flat disc). With the increase in thickness parameter (k = 2, 4), the circumferential stress is maximum at the external surface while the radial stress is maximum at the internal surface. From the figures drawn the disc with exponentially varying thickness (k = 2), a high angular speed is required for initial yielding at the internal surface as compared to flat disc and exponentially varying thickness for k = 4 onwards. It is concluded that the disk made of isotropic compressible material is on the safer side of the design as compared to disk of isotropic incompressible material as it requires higher increase in an angular speed to become fully plastic from its initial yielding.

Thakur P. et al. (2013) have obtained stresses for the elastic – plastic transition and fully plastic state has been derived for a thin rotating disc having variable thickness with shaft at different temperatures. It has been observed that in the

absence of thickness, the rotating disc made of incompressible material e.g. rubber with inclusion require a higher angular speed to yield at the internal surface as compared to a disc of compressible material e.g. copper, brass and steel and a much higher angular speed is required to yield with the increase in radii ratio. With the effect of variation thickness, reverse results and lesser angular speed are required to yield at the internal surface. With the introduction of thermal effects for k = 0, lesser angular speed is required to yield at the internal surface. When thickness varies (say k = 2.5, 5), reverse in the results. The radial stresses is maximum at the internal surface with no thickness, when thickness varies the circumferential stress is maximum at the internal surface.

Thakur P. et al. (2016) discussed the problem in Thermal creep transition stresses and strain rates in a circular Disc with shaft having variable density. Thermal effect increased value of radial stress at the internal surface of the rotating disc made of incompressible material as compared to tangential stress and this value of radial stress further much increases with the increase in angular speed as compared to without thermal effect. Strain rates have maximum values at the internal surface for compressible material.

Thakur P. et al. (2017) have analyzed thermal creep stress and strain analysis in Non-homogeneous Spherical shell by using Seth transition theory. With the effect of thermal effect, values of circumferential stress decrease at the external surface as well as internal surface of the spherical shell. It means that the temperature dependent materials minimize the possibility of fracture at the internal surface of the spherical shell.

Thakur P. et al. (2017) discussed the problem in elastic-plastic stress analysis in a spherical shell under internal pressure and steady state temperature by using Seth transition theory. It has been seen that circumferential stress has a maximum at the external surface of the spherical shell made of incompressible- as well as compressible materials. The thermal effect increases the values of stresses.

Thakur P. et al. (2017) analyzed the problem in elastic-plastic transition on rotating spherical shells in dependence of compressibility. It has been observed that Rotating shells made of the incompressible material are on the safer side of the design as compared to rotating shells made of compressible material. The effect of density variation has been discussed numerically on the stresses. With the effect of density variation parameter, rotating spherical shells start yielding at the internal surface with

the lower values of the angular speed for incompressible/compressible materials.

Thakur P. et al. (2017) analyzed the problem Creep strain rates analysis in cylinder under temperature gradient materials by using Seth's theory. Thermal effect increases the values of axial stress at the external surface of a circular cylinder for incompressible material as compared to compressible materials. With the introduction of thermal effects, the maximum value of strain rates occurs at the external surface for incompressible material as compared to the compressible materials

3. PROPOSED METHODOLOGY:

The deformation character has been considered to be linear in classical theory which is not accurate. The nonlinearity will be considered using the transition surface function and generalized strain measure to determine elastic-plastic and creep state. Seth has defined the generalized principal strain measure as,

$$e_{ii} = \int_0^{\overset{A}{e_{ii}}} \left[1 - 2\overset{A}{e}_{ii}\right]^{\frac{n}{2}-1} d\overset{A}{e}_{ii} = \frac{1}{n}\left[1 - \left(1 - 2\overset{A}{e}_{ii}\right)\right], (i = 1, 2, 3)$$

where n is the measure and $\overset{A}{e}_{ii}$ are the Almansi finite strain components.

Seth's defined transition function for elastic-plastic and creep state. When a deformable solid is subjected to an external loading system, it has been observed that the solid first deforms elastically. If the loading is continued, plastic flow may set in, and if continued further, it gives rise to time dependent continuous deformation known as creep deformation. It may be possible that a number of transition states may occur at the same critical point, then the transition function will have different values, and the point will be a multiple point, each branch of which will then correspond to a different state. In general, the material from elastic state can go over into (i) plastic state, or to (ii) creep state, or (iii) first to plastic state and then to creep and vice-versa, depending upon the loading. All these final states are reached through a transition state. In the plane stress condition, the transition can take place either through the principal stresses τ_{rr} or $\tau_{\theta\theta}$ becoming critical or through the principal stress difference $\tau_{rr} - \tau_{\theta\theta}$ becoming critical. Hence we have to consider the following three cases: (*a*) Transition through τ_{rr}, (*b*) Transition through $\tau_{\theta\theta}$, (*c*) Transition through $\tau_{rr} - \tau_{\theta\theta}$. For each transition point, we have to determine the stresses and strains corresponding to the above three cases.

4. OBJECTIVES OF STUDY:

The objective of research is to be present original results of mathematical modeling problems in mechanics of deformable solids. The mechanical process involving deformable solids are abundant in industry and play an important role in engineering systems and structures. They include a large variety of phenomena and therefore the need of mathematical models that can predict reliably the evolution of deformable bodies under the action of external loads was recognized a long time ago. Mathematical models used in solid mechanics are usually expressed in terms of partial differential equations, associated with various initial and boundary conditions. Their validity depends on how well the theoretical results they predict, agree with the results of reputed experiments. Indeed, the crucial modeling criterion is that a mathematical model should be able to describe those mechanical properties of the real systems that are under consideration with the desired accuracy. In some special cases, the solution is exact under the assumption made during mechanical and mathematical modeling.
Mathematical modeling in solid mechanics requires the following in general:

- Construction of mathematical model based on constitutive assumptions and thermodynamics/thermo-dynamical principles.
- Mathematical and numerical analysis of models. It is devoted to existence, uniqueness and convergence results for the continuous and discrete solutions.
- Construction of reliable and efficient algorithms for the numerical and experimental results.

5. THESIS OUTLINE:

In the proposed thesis an attempt will be made to study characterization of materials on elastic-plastic and creep stresses problems in the disc, cylinder and spherical shell under various conditions by using Seth's transition theory and generalized strain measure. The tentative chapter wise plan of the thesis has been proposed as under:

Chapter 1: Introduction

Chapter 2: Study for characterization of materials in disc under various conditions.

Chapter 3: Study for characterization of materials in cylinder under various conditions.

Chapter 4: study for characterization of materials in shell under various conditions.

Chapter 5: Conclusions and scope for further research.

6. RESEARCH PLAN SCHEDULE:

Research activity	**Progress in months**						
No. of Months	**0**	**6**	**12**	**18**	**24**	**30**	**36**
Course works, extended literature review	⇢						
Construction of theoretical framework/model		⇢					
Determination of stress and strain/ rates			⇢				
Result and discussion				⇢			
Publication					⇢		
Thesis writing						⇢	

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